**3D Fourier Transforms**

The inverse Fourier transform, recovering from is:

Also note how:

In Fourier space, differentiation becomes multiplication: this greatly simplifies numerical and analytical analysis.

3D and 2D Fourier Transforms are linear operators. This proof is trivial.

The subscript is the complex conjugate operation. . Fubini’s theorem [https://en.wikipedia.org/wiki/Fubini%27s\_theorem] allows us to change the order of integration arbitrarily.

**2D Fourier Transforms**

The 2D FT of the in-plane derivatives () of can be found as:

The out-of-plane derivative of can be found as:

The 2D FT of the out-of-plane derivative of can be found as:

The commutes with the and operators because the 2D FT and IFT have no dependence on .

**Discrete Fourier Transform**

Computationally, the Fourier Transform and Inverse Fourier Transform are implemented using the discrete Fourier transform. The sets and represents discrete points in Fourier and real space respectively.

The 3D discrete FT and IFT are implemented using the Fast Fourier Transform (FFT) algorithm and are given as:

Let us think of the indices as labeling points for the data sets. These indices have no physical meaning. The discrete Fourier space coordinates are:

The real space coordinates are:

The result of the discrete Fourier transform is a data set index by the Fourier space coordinates. Within this data set, at a given Fourier space coordinate lies the contribution of the given Fourier space coordinate to the real space data. is the Nyquist value. The discrete Fourier space coordinates stop at the Nyquist value because if the Nyquist value were exceeded, it would be aliased:

Essentially, any Fourier space coordinate greater than the Nyquist value is identical to a Fourier space coordinate within the Nyquist range. Physically, we can think of the Fourier transform as finding the sine wave frequencies that contribute to a signal. If the sampling between data points is not fine enough, we cannot resolve high frequencies…

The derivative in Fourier space is given as (DERIVE???):

We can equate our Discrete Fourier Transform space derivative with our continuous Fourier Transform derivative by defining .

Also note: where is the physical dimension of the dimension. Our formula uses to normalize the discrete derivative to real dimensions.